Numerical Studies of Recirculating Ducted Flows

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Abstract

A COMPUTATIONAL method for calculating steady flow in complex, two-dimensional ducts was developed, tested, and verified. The method was developed to permit accurate computations of turbulent flows at low speeds in arbitrary two-dimensional duct configurations. Particular effort was made to develop a method that could resolve velocity derivatives (e.g., $\partial u/\partial r$) because these terms, squared, are contained in the production terms for turbulent kinetic energy and its dissipation rate and hence strongly affect the accuracy of the turbulence transport model. Also, wall function based boundary condition subroutines were developed for vorticity, stream function, velocity components, species concentrations, temperature and/or energy, pressure, turbulent kinetic energy, and dissipation for complex wall geometries. At its present stage, the method has been verified against experimental data in simple and complex geometries based on a variety of transport models including the κ-ϵ turbulence transport model.

Nomenclature

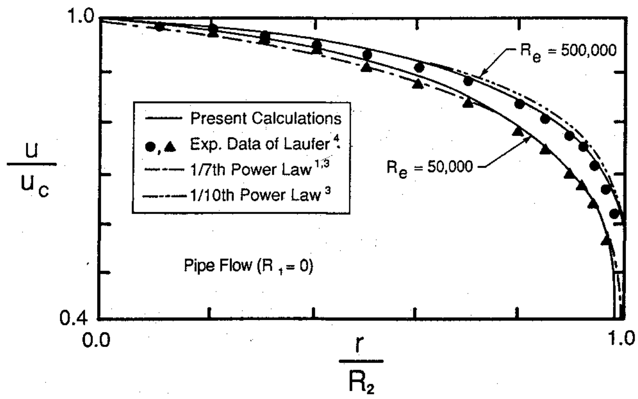
H = step height
r = radial coordinate in physical plane
R_1 = radius of inner wall
R_2 = radius of outer wall
R_e = flow mean Reynolds number based on the inlet diameter
u = axial velocity
u_c = centerline axial velocity
u_{max} = maximum axial velocity
x = axial coordinate in physical plane
x_r = reattachment length
x_{rc} = axial location of recirculating center
ϵ = dissipation rate of turbulent kinetic energy
κ = turbulent kinetic energy

Contents

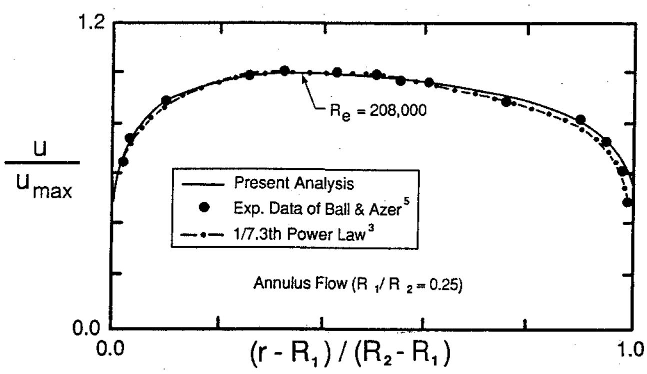
The computational formulation is based on the stream function/vorticity variables obtained from the full, variable density, variable viscosity vector form of the Navier-Stokes equation by taking the curl operation.[1,2] All derivatives of density and viscosity are retained and are included in the so-called "source term" for the vorticity transport equation that results. The stream-function equation results from the introduction of stream-function derivatives into the equation for the only nonzero component of vorticity in two-dimensional flows. The general formulation of the vorticity transport equation is valid for axisymmetric two-dimensional flows with axially symmetric swirl component. Thus, there are up to three components of velocity that can be obtained with this computational method. The three velocity components are also taken into account in the general transport equations that are solved to predict the spatial distributions of species, temperature and/or energy, pressure, and turbulence kinetic energy and its dissipation rate. The pressure field is obtained as the concurrent solution of a Poisson field equation with mean and turbulent velocity component source terms.

The solution procedure is based on a finite-difference solution procedure for fully elliptic partial differential equations applied to the governing system of equations after they have first been transformed into a general nonorthogonal coordinate system. At each point of the computational domain, including all boundary points, a Jacobian exists that can transform the computed results back into the physical coordinate system.

Considerable effort was made to develop a wall function library that the computational methodology can call on for establishing boundary values for all dependent variables. These wall functions include inlet velocity profile functions for "fully developed" inlet flows, including both laminar and turbulent flows. From these profile functions, inlet distributions for stream function, vorticity, species, temperature, turbulent kinetic energy, etc., are established. Outlet or exit flow boundary conditions are usually just the "fully developed outlet flow" conditions ($\partial/\partial x = 0$ or



a) In a pipe.



b) In an annulus.

Fig. 1 Fully developed turbulent velocity profiles.

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$\partial^2/\partial x^2 = 0$). The other wall functions are profile families for laminar and/or turbulent boundary-layer flows, including those with both longitudinal and azimuthal velocity components.

Some of the results of the computations made with this solution procedure are presented in Figs. 1-3. Figure 1 shows data and calculated results of turbulent flow in a pipe[1,3] and in an annulus.[3] This figure shows that for simple high

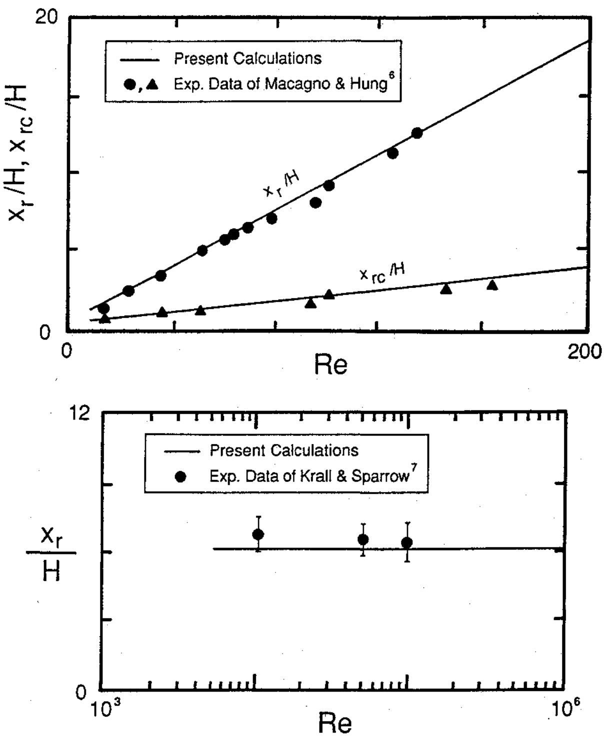

Fig. 2 Reattachment length x_r and location of the recirculating center x_{rc} for a symmetric sudden expansion.

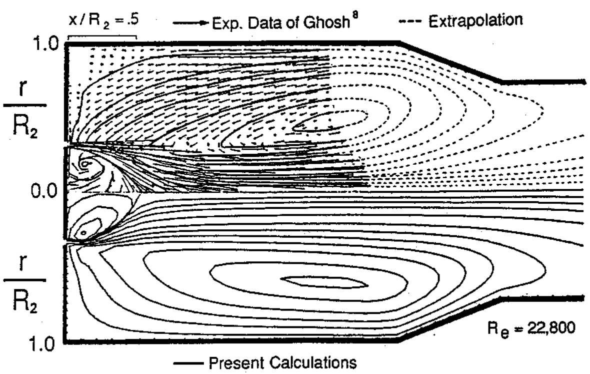

Fig. 3 Comparison of numerical calculations to data of UTSI cold-flow experiments.

Reynolds number flows the method gets accurate results. Figure 2 shows something interesting. We computed the reattachment distance of a separating ducted flow downstream of a sudden pipe expansion as a function of the Reynolds number of the flow. Experiment and our computations show that above a certain Reynolds number, 5×10^3, in our nomenclature, the reattachment distance is fixed. We interpreted this as evidence of the accuracy of our computational method. Finally, Fig. 3 shows some computed streamlines superimposed on a measured vector velocity field. The measurements were taken in an axisymmetric, isothermal combustor model at high Reynolds number (turbulent) conditions. An annular slit jet is issuing into the combustor, driving both inner and outer recirculation zones. Detailed comparisons have been made between measured and predicted velocity fields with excellent comparisons. Figure 3 is presented to illustrate with visual impact that the code appears to be providing an acceptable numerical simulation of the real flow.

Conclusions

Although the stream-function/vorticity approach to numerically modeling fluid flow is no longer popular for finite-difference solutions, the authors have developed a version that gives highly satisfactory results in complex, two-dimensional duct geometries. Both laminar and turbulent flows with variable density and variable viscosity have been computed with good results.[1-3] The program is currently being extended to include particle transport, two-phase effects such as evaporation and chemistry. The code is available to the public in its present form.

Acknowledgment

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